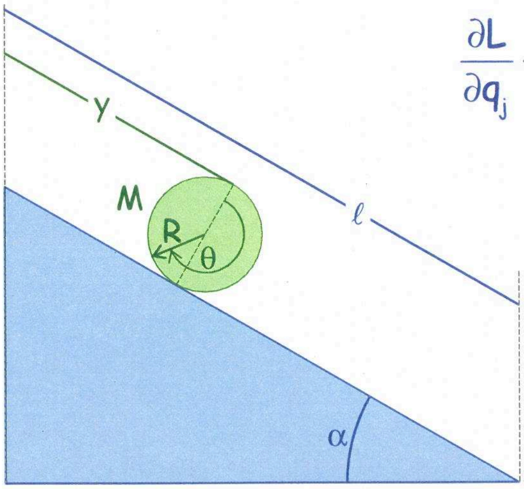
4) TM5 Example 7.9 ... using NSL

Using Lagranges equations with undetermined multipliers to analyze a disk rolling down a fixed wedge,



$$\frac{\partial L}{\partial q_j} - \frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} + \sum_k \lambda_k \frac{\partial f_f}{\partial q_j} = 0 \quad \text{and} \quad Q_j = -\sum_k \lambda_k \frac{\partial f_k}{\partial q_j}$$

We found the generalized forces to be

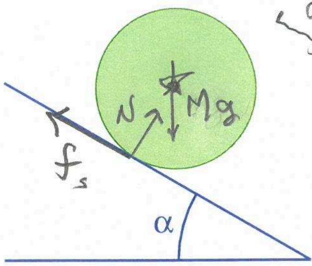
$$Q_y = -\frac{Mg\sin\alpha}{3} \quad \text{and} \quad Q_\theta = +\frac{MgR\sin\alpha}{3}$$

Apply NSL in linear and angular forms to the same problem to determine what these forces are. What is the meaning of the 3?

Apply NSL: $\sum F_\perp = M\cancel{a_\perp}^{0}$ $\qquad \sum F_\parallel = Ma_\parallel$

$N - Mg\cos\alpha = 0$ $\qquad \boxed{Mg\sin\alpha - f_s = Ma}$

$N = Mg\cos\alpha$



Also for rotation about the center

$Rf = I\alpha$ ← Required for rolling

$\cancel{R}f = \left(\frac{1}{2}MR^{\cancel{2}}\right)\left(\frac{a}{\cancel{R}}\right)$

$$\boxed{f = \frac{1}{2}Ma}$$

Substitute this into $\sum F_\parallel$ equation

$Mg\sin\alpha - \frac{1}{2}\cancel{M}a = \cancel{M}a \Rightarrow g\sin\alpha = \frac{3}{2}a$

Now write the friction

$f = \frac{1}{2}Ma = \frac{1}{\cancel{2}}M\left(\frac{\cancel{2}}{3}g\sin\alpha\right) = \boxed{\frac{1}{3}Mg\sin\alpha = Q_y}$ ← Friction!

The sign indicates f is opposite $\vec{a}$ which we know a priori.

The torque is

$\tau_f = Rf = \boxed{\frac{1}{3}MgR\sin\alpha = Q_\theta}$

$\sum$ Torque!